Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Paper reference **9MA0/32**

Mathematics

Advanced

PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

P72131A

Q:1/1/1/1/

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Pearson

1. [*In this question, position vectors are given relative to a fixed origin.*]

At time t seconds, where $t > 0$, a particle P has velocity $\mathbf{v}\,\mathrm{m\,s^{-1}}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of P at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of t, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of P at time t seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of P is $(\mathbf{i} - 4\mathbf{j})$ m.

(c) Find the position vector of P at time $t = 1$ second.

(4)

$V = \begin{pmatrix} 3t^2 \\ -6t^{\frac{1}{2}} \end{pmatrix}$

a) When $t = 2$: $V = \begin{pmatrix} 3(2)^2 \\ -6\sqrt{2} \end{pmatrix} = \begin{pmatrix} 12 \\ -6\sqrt{2} \end{pmatrix}$

speed = mag of velocity

$= \sqrt{(12)^2 + (-6\sqrt{2})^2}$

$= \sqrt{144 + 36(2)}$

$= \sqrt{144 + 72}$

$= \sqrt{216} = 14.7\ \mathrm{ms^{-1}}$

b) acceleration is the derivative of velocity

$a = \frac{dv}{dt} = \begin{pmatrix} 6t \\ -3t^{-\frac{1}{2}} \end{pmatrix}$

$= 6t\underline{i} - \frac{3}{\sqrt{t}}\underline{i}$

Question 1 continued

c) $t=4$: position vector $=\begin{pmatrix}1\\-4\end{pmatrix}$

Displacement $=\int(\text{velocity})dt$

We just integrate each component

Way 1:

Displacement $=\begin{pmatrix}\int 3t^2\,dt\\ \int -6t^{\frac{1}{2}}\,dt\end{pmatrix}=\begin{pmatrix}\frac{3t^3}{3}+c_1\\ -\frac{6t^{\frac{3}{2}}}{\frac{3}{2}}+c_2\end{pmatrix}$

$=\begin{pmatrix}t^3+c_1\\ -4t^{\frac{3}{2}}+c_2\end{pmatrix}$

Use when $t=4$, position $=\begin{pmatrix}1\\-4\end{pmatrix}$: use this to find c_1 and c_2

$\begin{pmatrix}1\\-4\end{pmatrix}=\begin{pmatrix}4^3+c_1\\ -4(4)^{\frac{3}{2}}+c_2\end{pmatrix}$

equate components

$1=64+c_1$ $\qquad$ $-4=-4(8)+c_2$

$c_1=-63$ $\qquad$ $c_2=28$

sub these back into the displacement

so position $=\begin{pmatrix}t^3-63\\ -4t^{\frac{3}{2}}+28\end{pmatrix}$

(Total for Question 1 is 8 marks)

Question 1 continued

Sub in $t=1$:

Position $= \begin{pmatrix} 1-63 \\ -4+28 \end{pmatrix} = \begin{pmatrix} -62 \\ 24 \end{pmatrix}$

$-62\underline{i} + 24\underline{j}$

Way 2:

Displacement $= \begin{pmatrix} \int_1^4 3t^2 \, dt \\ \int_1^4 -6t^{\frac{1}{2}} \, dt \end{pmatrix}$

$= \begin{pmatrix} 63 \\ -28 \end{pmatrix}$

Displacement = position at time $t=4$ − position at time $t=1$

$\begin{pmatrix} 63 \\ -28 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix} - \begin{pmatrix} 63 \\ -28 \end{pmatrix} = \begin{pmatrix} -62 \\ 24 \end{pmatrix} = -62\underline{i} + 24\underline{j}$

(Total for Question 1 is 8 marks)

Note:
Doesn't matter which direction we use for friction. The maths from resolving will tell us if correct. If negative answer for F it means going opposite to direction chosen

2.

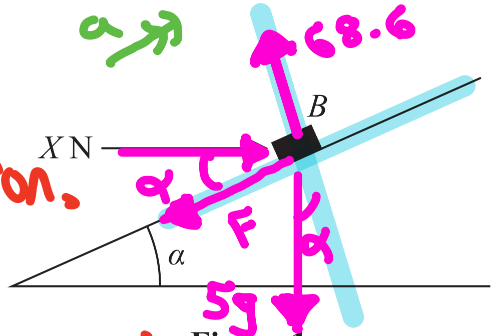
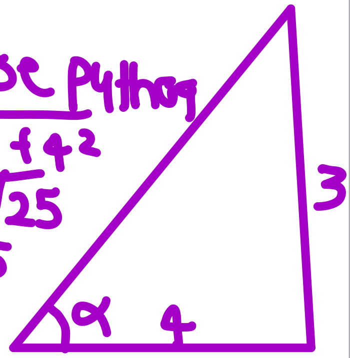


Figure 1

Use Pythag
$\sqrt{3^2+4^2}$
$=\sqrt{25}$
$=5$



A rough plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

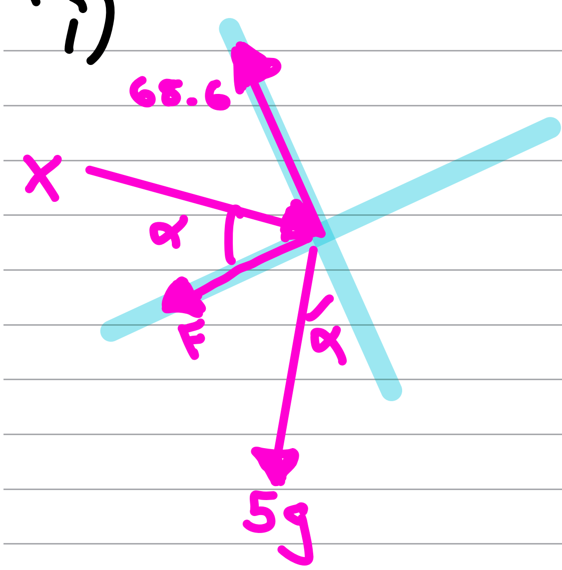
The horizontal force of magnitude X newtons is now removed and B moves down the plane.

Given that the coefficient of friction between B and the plane is 0.5

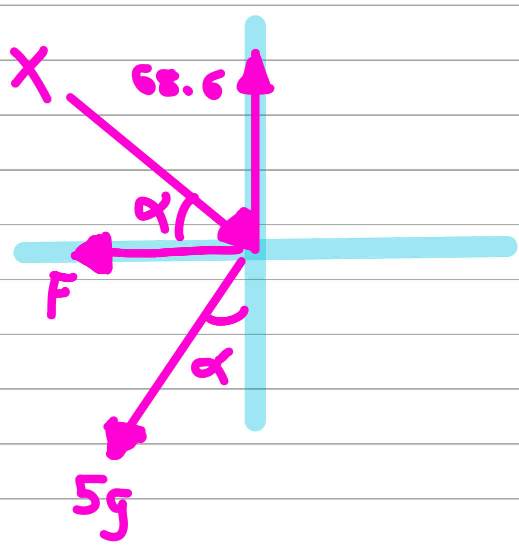
(b) find the acceleration of B down the plane.

(6)

a) i)

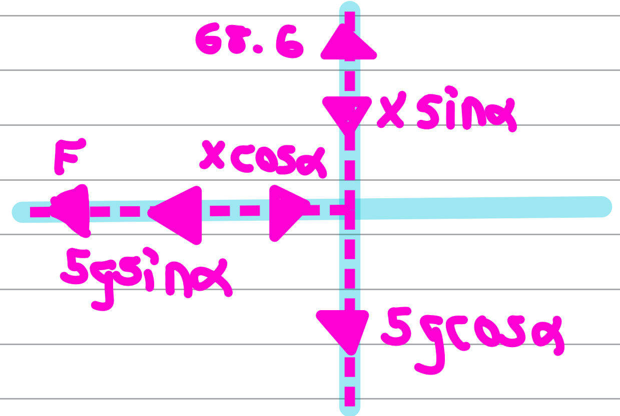


Twist axis
⇒



Question 2 continued

resolve diagonal forces into horizontal & vertical components $\Rightarrow$



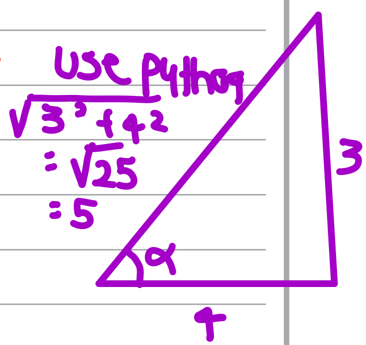
Let's resolve in the horizontal and vertical directions since we have forces in these directions:

- Resolve perpendicular:

$\uparrow$: Follow the template $F = ma$

$R - X\sin\alpha - 5g\cos\alpha = 5(0)$ ↘ no movement in this direction

given $\tan\alpha = \frac{3}{4}$: Build triangle



$68.6 - X\left(\frac{3}{5}\right) - 5g\left(\frac{4}{5}\right) = 0$ ①

$\frac{3}{5}X = 68.6 - 4g$

$\frac{3}{5}X = 29.4$

$X = 49$ N

- Resolve Parallel:

$\rightarrow$: Follow the template $F = ma$

$X\cos\alpha - 5g\sin\alpha - F = 5(0)$ ↘ $= 0$ since in equilibrium

$X\left(\frac{4}{5}\right) - 5g\left(\frac{3}{5}\right) - F = 0$

$49\left(\frac{4}{5}\right) - 3g - F = 0$

$F = -3g + \frac{196}{5} = 9.8$ Magnitude is 9.8 N

Question 2 continued

ii) It acts down the plane (we know this since we chosen down the plane as our positive sense and our answer for F was positive which means going in same direction to positive sense chosen.

b) Re-resolve perpendicular & Parallel again but this time without the force X

• Perpendicular:

$\uparrow$: $R - 5g\cos\alpha = 5(0)$

$R = 5g\cos\alpha$

$= 5g\left(\frac{4}{5}\right)$

$= 4g$

$= 39.2$ N

• Parallel: Take down the plane as positive sense now

$\swarrow$: $5g\sin\alpha - F = 5a$ → we now have acceleration

$5g\left(\frac{3}{5}\right) - F = 5a$

$3g - F = 5a$ ①

• We also know $F = \mu R$

$= 0.5(39.2) = 19.6$ N

① becomes $3g - 19.6 = 5a$

$5a = 9.8$

$a = 1.96 \text{ ms}^{-2}$

3. [*In this question*, **i** *and* **j** *are horizontal unit vectors*.]

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\,\text{N}$ and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\,\text{N}$, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

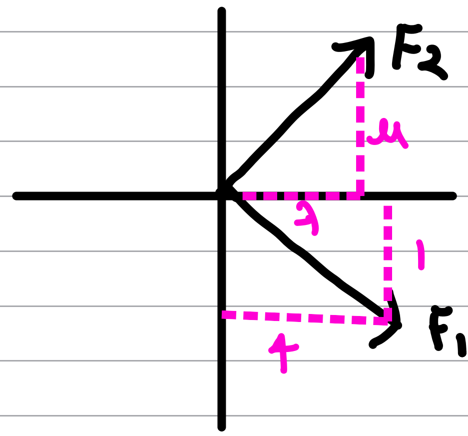
At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

$F_1 = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $F_2 = \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$



Resultant $= \begin{pmatrix} \lambda \\ \mu \end{pmatrix} + \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} \lambda + 4 \\ \mu - 1 \end{pmatrix}$

Moves in the direction means parallel to

so $\begin{pmatrix} \lambda + 4 \\ \mu - 1 \end{pmatrix}$ is paralel to $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ hence a multiple of. This means

$\begin{pmatrix} \lambda + 4 \\ \mu - 1 \end{pmatrix} = k\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} \lambda + 4 \\ \mu - 1 \end{pmatrix} = \begin{pmatrix} 3k \\ k \end{pmatrix}$

equate components

$\lambda + 4 = 3k \Rightarrow k = \frac{\lambda+4}{3}$

$\mu - 1 = k$

Both expressions are equal to k so we can set them equal

$\frac{\lambda+4}{3} = \mu - 1$

$\lambda + 4 = 3\mu - 3$

$\lambda - 3\mu + 7 = 0$ as required

b) $\lambda = 2$:

$2 + 4 = 3\mu - 3$

$6 = 3\mu - 3$

$\mu = 3$

Resultant force $= \begin{pmatrix} \lambda + 4 \\ \mu - 1 \end{pmatrix} = \begin{pmatrix} 2+4 \\ 3-1 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

Plug into $F = ma$:

resultant mass acceleration

$\begin{pmatrix} 6 \\ 2 \end{pmatrix} = 4\begin{pmatrix} x \\ y \end{pmatrix}$

$\begin{pmatrix} 6 \\ 2 \end{pmatrix} = \begin{pmatrix} 4x \\ 4y \end{pmatrix}$

equate coefficients

Question 3 continued

$6 = 4x$ and $2 = 4y$
$x = 1.5$ $y = 0.5$

$a = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$

In order to find the length of AB, we need the displacement, so we need to integrate the acceleration

$V = \begin{pmatrix} \int 1.5\,dt \\ \int 0.5\,dt \end{pmatrix}$
$V = \begin{pmatrix} 1.5t + c_1 \\ 0.5t + c_2 \end{pmatrix}$

When $t = 0$, $V = 0$

$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1.5t + c_1 \\ 0.5t + c_2 \end{pmatrix}$

equating components:
$0 = 1.5t + c_1$ and $0 = 0.5t + c_2$
$c_1 = 0$ $c_2 = 0$

$\Rightarrow V = \begin{pmatrix} 1.5t \\ 0.5t \end{pmatrix}$

Integrate again to get s

$s = \begin{pmatrix} \frac{1.5t^2}{2} + c_3 \\ \frac{0.5t^2}{2} + c_4 \end{pmatrix}$

$s = \begin{pmatrix} 0.75t^2 + c_3 \\ 0.25t^2 + c_4 \end{pmatrix}$

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At A: $t=0$, $s=0$

$$\begin{pmatrix}0\\0\end{pmatrix}=\begin{pmatrix}c_3\\c_4\end{pmatrix}$$

so $c_3=0$, $c_4=0$

$$s=\begin{pmatrix}0.75t^2\\0.25t^2\end{pmatrix}$$

At B: $t=4$

$$s=\begin{pmatrix}0.75(4^2)\\0.25(4^2)\end{pmatrix}=\begin{pmatrix}12\\4\end{pmatrix}$$

$$AB=\begin{pmatrix}12\\4\end{pmatrix}$$

Length of $AB=\sqrt{12^2+4^2}=\sqrt{160}=4\sqrt{10}$

$=12.6\text{ m}$

(Total for Question 3 is 9 marks)

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4.

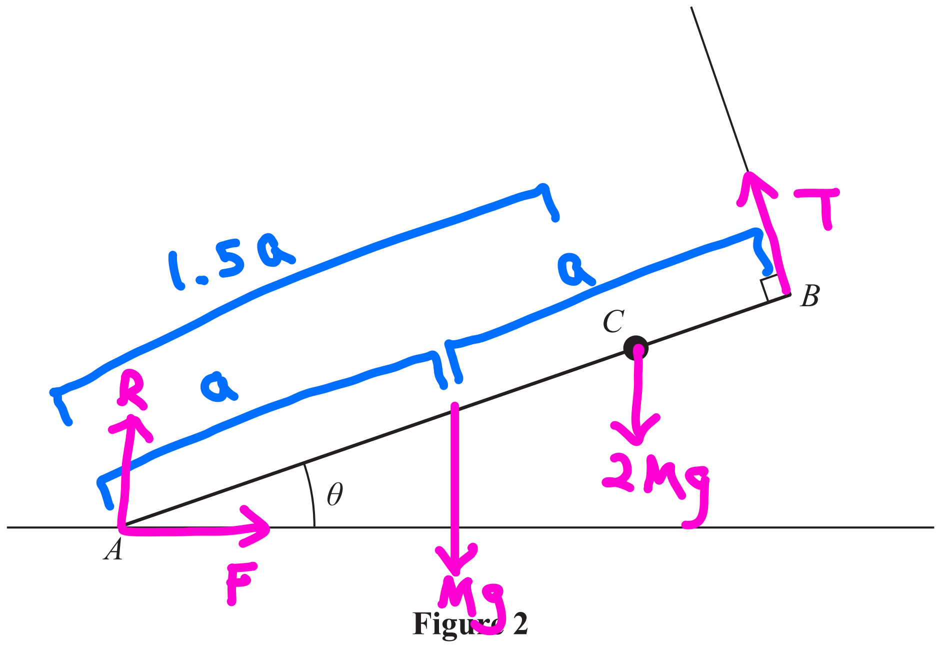


Figure 2

A uniform rod AB has mass M and length $2a$

A particle of mass $2M$ is attached to the rod at the point C, where $AC = 1.5a$

The rod rests with its end A on rough horizontal ground.

The rod is held in equilibrium at an angle θ to the ground by a light string that is attached to the end B of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

(a) Explain why the frictional force acting on the rod at A acts horizontally to the right on the diagram.

(1)

The tension in the string is T

(b) Show that $T = 2Mg\cos\theta$

(3)

Given that $\cos\theta = \dfrac{3}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at A is $\dfrac{57Mg}{25}$

(3)

The coefficient of friction between the rod and the ground is μ

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \dfrac{8}{19}$

(4)

Question 4 continued

a) The frictional force acts in the opposite direction to the direction that the rod will move. In this case the string exerts a force to the left so the frictional force acts to the right in order to maintain equilibrium.

b) Let's draw an axis for reference

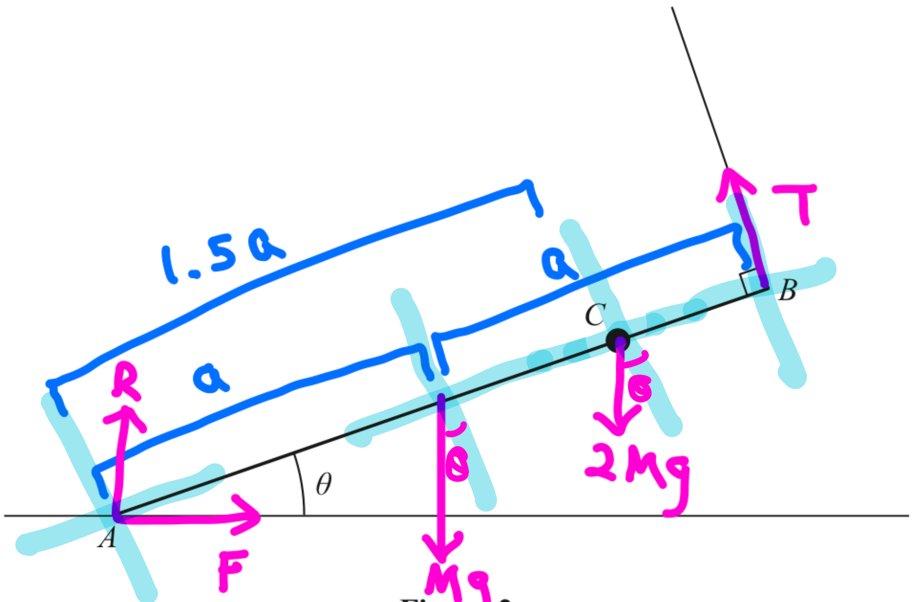
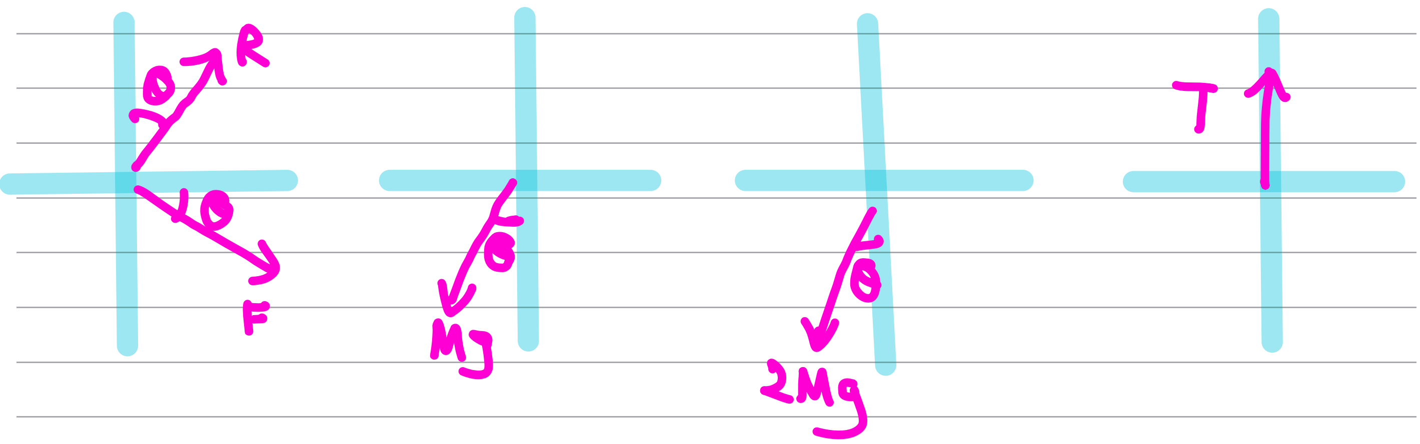


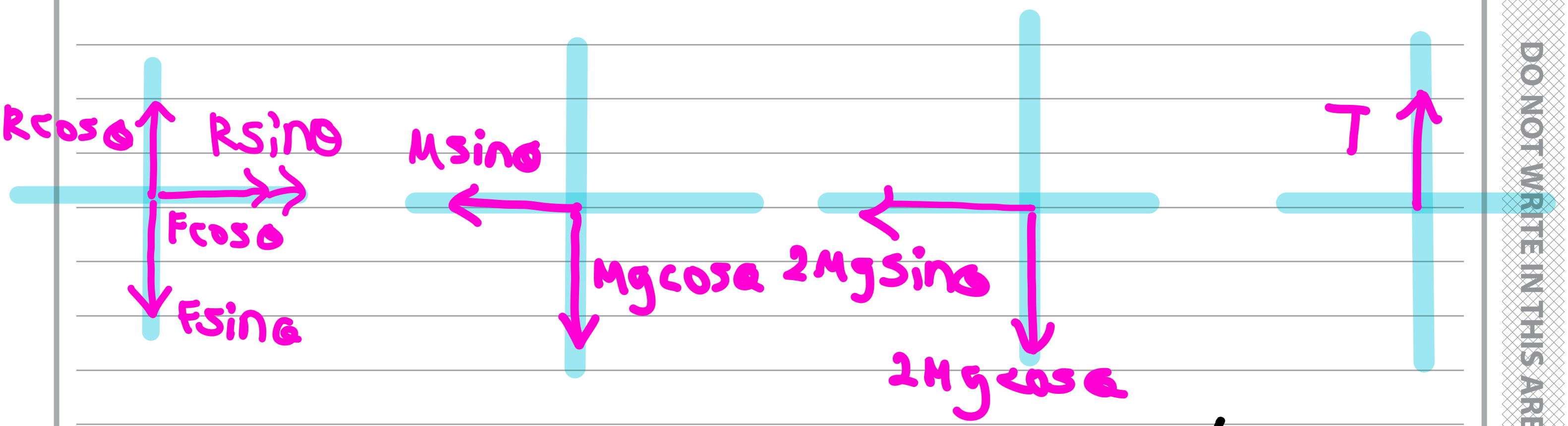
Figure 2

Twist the axes

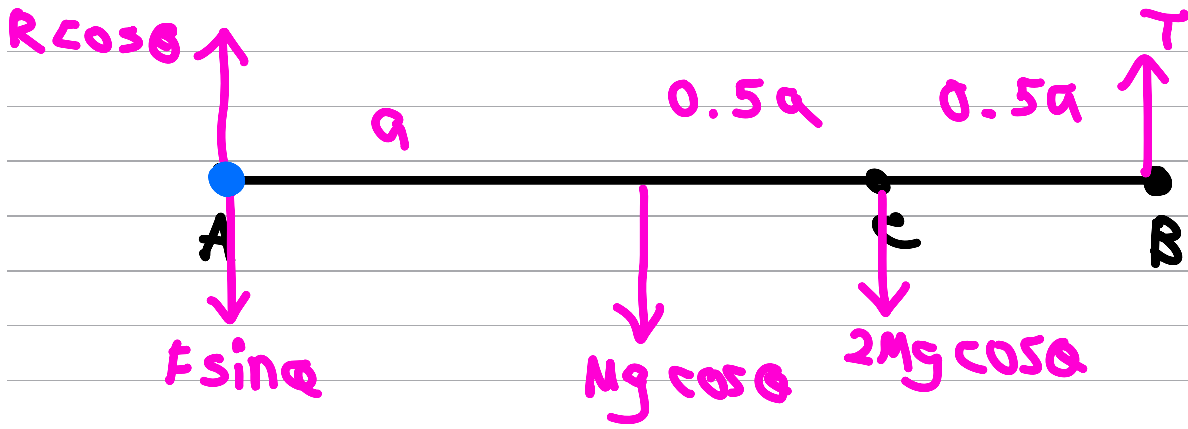


Resolve any diagonal forces

Question 4 continued



We only consider the perpendicular forces when taking moments. Let's put these on a flat Rod



$M(A)\circlearrowright: Mg\cos\theta(a) + 2Mg\cos\theta(1.5a) - T(2a) = 0$

$\Rightarrow Mga\cos\theta + 3Mga\cos\theta - 2aT = 0$

$\Rightarrow 4Mga\cos\theta - 2aT = 0$

$\Rightarrow a(4mg\cos\theta - 2T) = 0$

$a \neq 0,\ 4mg\cos\theta - 2T = 0$

$T = 2mg\cos\theta$ as required

Question 4 continued

c) Let's draw an axis for reference and resolve like we normally do

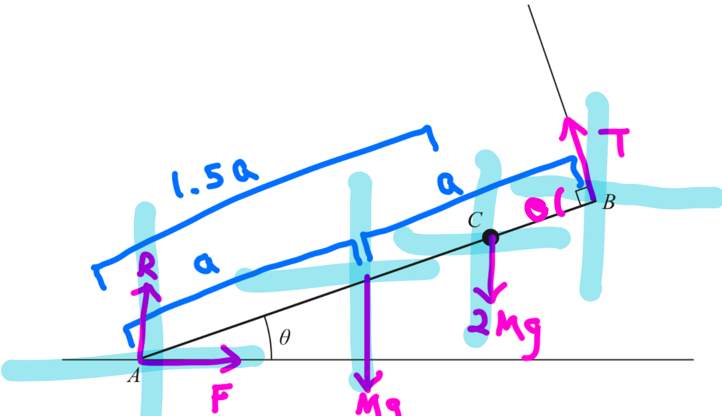
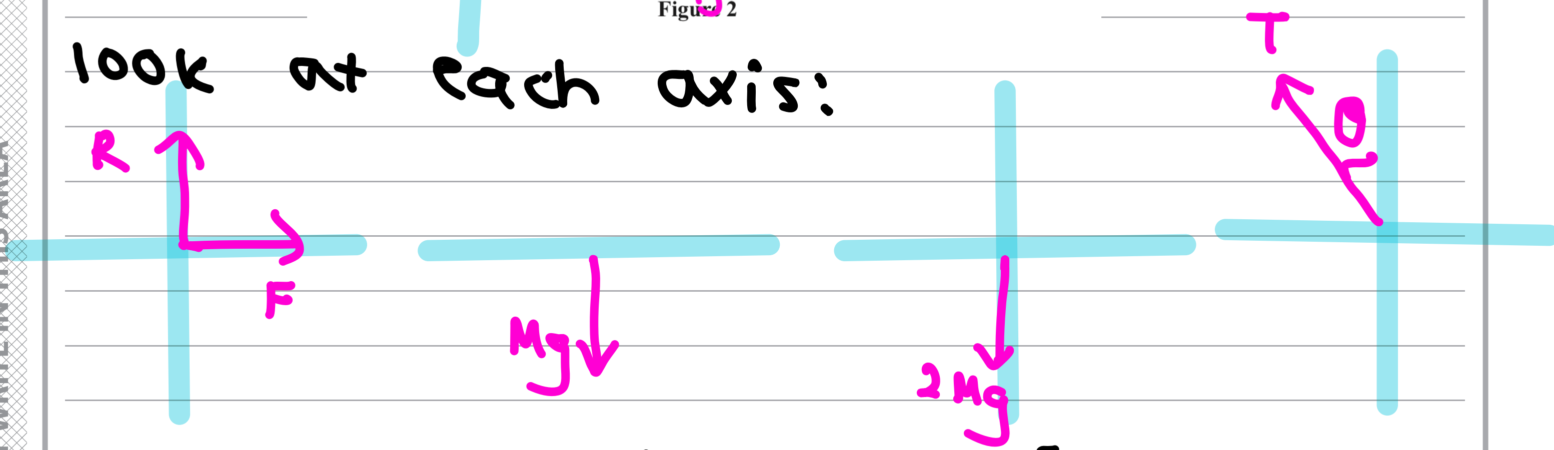


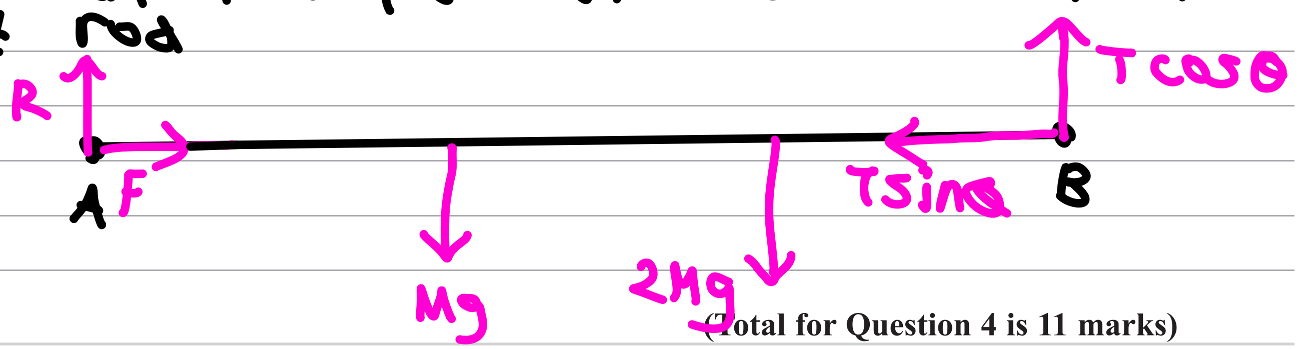
Figure 2

look at each axis:



Resolve any diagonal forces

We can now put all forces on a flat rod



(Total for Question 4 is 11 marks)

Question 4 continued

Resolve like usual (not taking moments):

$\uparrow$: $R - Mg - 2Mg + T\cos\theta = 0$

$R - 3Mg + T\cos\theta = 0$ ①

$\rightarrow$: $F - T\sin\theta = 0$ ②

Let's use ①

$R - 3Mg + T\cos\theta = 0$

$R - 3Mg + \underbrace{T}_{=2Mg\cos\theta \text{ from b)}}\cos\theta = 0$

$R = 3Mg - 2Mg\cos^2\theta$

Also given that $\cos\theta = \frac{3}{5}$

$R = 3Mg - 2Mg\left(\frac{3}{5}\right)^2$

$R = 3Mg - 2Mg\left(\frac{9}{25}\right)$

$R = 3Mg - \frac{18}{25}Mg$

$R = \frac{57}{25}Mg$

d) Let's use $F - T\sin\theta = 0$ from ②

$F = \underbrace{T}_{=2Mg\cos\theta \text{ from b)}}\sin\theta = 2Mg\cos\theta\sin\theta = 2Mg\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}Mg$

(Total for Question 4 is 11 marks)

P 7 2 1 3 1 A 0 1 5 2 0

Question 4 continued

$F = \mu R$

$\frac{24}{25} \cancel{M}g = \mu\left(\frac{57}{25}\cancel{M}g\right)$

$\frac{24}{25} = \mu\left(\frac{57}{25}\right)$

$\mu = \dfrac{\frac{24}{25}}{\frac{57}{25}} = \frac{8}{19}$

(Total for Question 4 is 11 marks)

5.

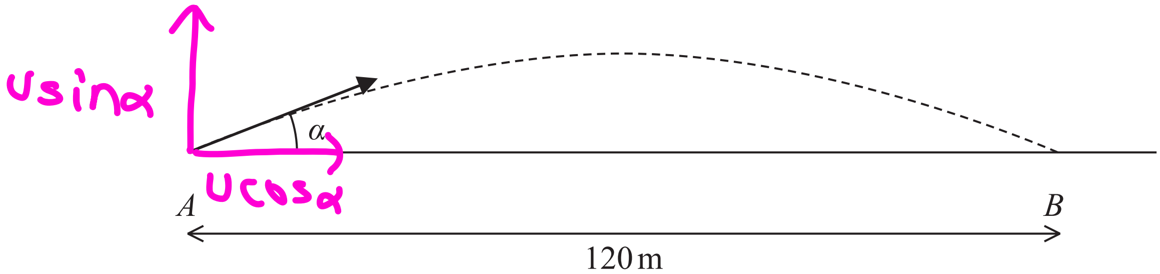


Figure 3

A golf ball is at rest at the point A on horizontal ground.

The ball is hit and initially moves at an angle α to the ground.

The ball first hits the ground at the point B, where $AB = 120$ m, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is U m s^{-1}

Using this model,

(a) show that $U^2 \sin\alpha \cos\alpha = 588$

(6)

The ball reaches a maximum height of 10 m above the ground.

(b) Show that $U^2 = 1960$

(4)

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from A to B, is now modelled as that of a particle whose initial speed is V m s^{-1}

This refined model is used to calculate a value for V

(c) State which is greater, U or V, giving a reason for your answer.

(1)

(d) State one further refinement to the model that would make the model more realistic.

(1)

a) horizontal

$s = 120$

$u = U\cos\alpha$

$v =$

$a = 0$

$t =$

$s = ut + \frac{1}{2}at^2$

$120 = U\cos\alpha\, t + 0$

$t = \dfrac{120}{U\cos\alpha}$

vertical

$s = 0$

$u = U\sin\alpha$

$v =$

$a = -9.8$

$t = \dfrac{120}{U\cos\alpha}$

$s = ut + \frac{1}{2}at^2$

$0 = \cancel{U}\sin\alpha\left(\dfrac{120}{\cancel{U}\cos\alpha}\right) + \frac{1}{2}(-9.8)\left(\dfrac{120}{U\cos\alpha}\right)^2$

Question 5 continued

$$0 = 120\frac{\sin\alpha}{\cos\alpha} - 4.9\left(\frac{14400}{U^2\cos^2\alpha}\right)$$

$$120\frac{\sin\alpha}{\cos\alpha} = \frac{70560}{U^2\cos^2\alpha}$$

$$120\sin\alpha = \frac{70560}{U^2\cos\alpha}$$

$$120U^2\sin\alpha\cos\alpha = 70560$$

$$\boxed{U^2\sin\alpha\cos\alpha = 588}$$ as required

b) consider vertical again:

$s = 10$
$u = U\sin\alpha$
$v = 0$ (greatest height)
$a = -9.8$
$t =$

$$v^2 = u^2 + 2as$$

$$0 = (U\sin\alpha)^2 - 2(9.8)(10)$$
$$U^2\sin^2\alpha = 196 \quad (1)$$

We also know $U^2\sin\alpha\cos\alpha = 588$ from a) (2)

Do $\frac{(1)}{(2)}$: $\frac{\cancel{U^2}\sin^2\alpha}{\cancel{U^2}\sin\alpha\cos\alpha} = \frac{196}{588}$

Note: We could also have re-arranged both (1) and (2) for U^2 OR $\sin\alpha$ and set them equal

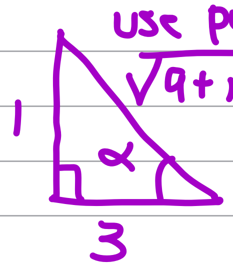
$$\frac{\sin\alpha}{\cos\alpha} = \frac{196}{588}$$

$$\tan\alpha = \frac{1}{3}$$

Question 5 continued

Build a triangle to find $\sin\alpha$ so that we can plug into ① which is $U^2\sin^2\alpha = 196$

use pythag $\sqrt{9+1} = \sqrt{10}$



$\sin\alpha = \frac{1}{\sqrt{10}}$

① becomes $U^2\left(\frac{1}{\sqrt{10}}\right)^2 = 196$

$$U^2\left(\frac{1}{10}\right) = 196$$

$U^2 = 1960$ as required

c) V is greater. In order to travel the same distance now that air resistance is considered, the initial velocity U must be greater

d) • Take into account the size and spin of the ball.

• model the ball as a particle

• use a more accurate value for gravity